LAUR-99-4581

Prospects of calculating ϵ_K and ϵ' from lattice QCD

Rajan Gupta

Theoretical Division, MS-B285, Los Alamos National Laboratory,
Los Alamos, NM 87545, USA.

February 1, 2008

Abstract

An overview of lattice results for the strange quark mass, B_K, B_6, B_7, and B_8 is presented. I give my assessment of the reliability of various estimates and prospects for future improvements.

1 Introduction

The status of CP violation in Kaon decays is

$$\begin{aligned} \epsilon &= (2.280 \pm 0.013)10^{-3} e^{i\pi/4} \\ \mathrm{Re}(\epsilon'/\epsilon) &= (21.2 \pm 4.6)10^{-4} \end{aligned} \tag{1}$$

where I have taken the most recent world average of ϵ'/ϵ from M. Sozzi's talk. The experimental errors on ϵ'/ϵ will decrease once KTeV and NA48 collaborations analyze their full data set, providing us with a unique opportunity to test the standard model. The spotlight is, therefore, now on theory: can one reconcile the two measurements with the predictions of the standard model in which both parameters are governed by the single phase in the CKM matrix, or are these results a window to new physics?

The standard model predictions, evaluated using the effective weak Hamiltonian defined at scale μ, are summarized by Buras in his talk:

$$\begin{aligned} \epsilon &= \mathrm{Im}\lambda_t C_\epsilon \hat{B}_K e^{i\pi/4} \{ \mathrm{Re}\lambda_c \left[\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t)\right] + \mathrm{Re}\lambda_t \eta_2 S_0(x_t) \} \\ \epsilon' &= \frac{i e^{i(\pi/4 + \delta_2 - \delta_0)}}{\sqrt{2}} \frac{\mathrm{Im}A_2}{\mathrm{Re}A_0} \left[1 - \epsilon^2\right] \\ &= \mathrm{Im}\lambda_t \frac{G_F e^{i(\pi/4+\delta_2-\delta_0)}}{2\mathrm{Re}A_0} \left[\omega \sum_i y_i \langle Q_i \rangle_0 (1 - \Omega_{\eta+\eta'}) - \sum_i y_i \langle Q_i \rangle_2 \right] \end{aligned} \tag{2}$$

where $C_\epsilon = 3.78 \times 10^4$, $\omega = \mathrm{Re}A_2/\mathrm{Re}A_0 \approx 1/22$, $\Omega_{\eta+\eta'}$ is the isospin breaking contribution, y_i are the Wilson coefficients, $\langle Q_i \rangle_I = \langle (\pi\pi)_I | Q_i | K \rangle$, and the sum is over all the 4-fermion operators that contribute. Also, I use the convention $\mathrm{Im}A_0 = 0$ and in the last expression neglect the term proportional to ϵ^2. As expected, both quantities are proportional to $\mathrm{Im}\lambda_t \equiv \mathrm{Im}V_{td}V_{ts}^* = A^2\lambda^5\eta$ in the Wolfenstein parameterization.

The equation for ϵ provides a parabolic constraint in the $\rho - \eta$ plane provided $\hat{B}_K$ and $|V_{cb}|$ are known. Alternately, a precise determination of $\hat{B}_K$ and $|V_{cb}|$ would fix $\mathrm{Im}\lambda_t$, and the measurements of ϵ'/ϵ could be used to look for new physics. Note that a larger value of $\hat{B}_K$ implies smaller $\mathrm{Im}\lambda_t$, and consequently smaller ϵ'.

For ϵ' we follow the work of Buras *et al.* [1] who use the relations between the various $\langle Q_i \rangle$ and make maximal use of experimental input. Then

$$\left| \frac{\epsilon'}{\epsilon} \right| = \mathrm{Im}\lambda_t \Big\{ c_0 + (c_6 B_6^{1/2} + c_8 B_8^{3/2}) \left(\frac{158 \text{ MeV}}{m_d + m_s} \right)^2 \Big\} \tag{3}$$

For $\mu = m_c$, $\Lambda_{QCD}^{(4)} = 325 MeV$, and central values of the other parameters, Buras *et al.* [1] get in the NDR scheme

$$\begin{aligned} \mathrm{Im}\lambda_t &\approx 1.29 \times 10^{-4}\,, \\ c_0 &\approx -1.4\,, \\ c_6 &\approx +7.9\,, \\ c_8 &\approx -4.1\,. \end{aligned} \tag{4}$$

From Eqs. 3,4 it is clear that there is a strong cancellation between the $\Delta I = 1/2$ (dominated by QCD penguin Q_6) and $\Delta I = 3/2$ (dominated by electroweak penguin Q_8) operators, and the value of the strange quark mass plays a crucial role. For $B_6 = B_8 = 1$ (vacuum saturation approximation (VSA) values currently used as inputs), one needs $m_s + m_d = 70$ MeV at $\mu = m_c$ instead of 158 MeV to get ϵ'/ϵ to $\approx 23 \times 10^{-4}$. A more likely scenerio is an enhancement of B_6, a suppression of B_8 and the quark masses, and/or a conspiracy of all other input parameters. It is therefore important to determine all three quantities, m_s, B_6, and B_8, accurately in order to resolve whether or not the SM predicts the observed value of ϵ'.

There are two omissions from my subsequent discussion of lattice calculations. First, I defer to T. Blum's talk for lattice results using domain wall fermions (DWF). Second, G. Martinelli has proposed analyzing B-parameters without introducing a dependence on $m_s + m_d$. Recall that the dependence on $m_s + m_d$ is introduced because in VSA $\langle O_6 \rangle, \langle O_8 \rangle \propto |\langle 0|\mathcal{P}|K\rangle|^2 = 4M_K^4 f_K^2/(m_s+m_d)^2$. Using B parameters as commonly defined has certain numerical advantages, but it does shift the scale dependence of the operator into a new quantity $(m_s + m_d)$. Which approach is

Fermion Action	Z	β	$B_K(\overline{MS}, 2\ \mathrm{GeV})$
Staggered [2]	1-loop	$6.0-6.4^*$	0.62(2)(2)
Staggered [3]	1-loop	$5.7-6.65^*$	0.628(42)
Staggered [4]	1-loop	$5.7-6.2^*$	0.573(15)
Wilson [5]	1-loop TI	6.0	0.74(4)(5)
Wilson [6]	1-loop & χWI	$5.9-6.5^*$	0.69(7)
Clover [7]	1-loop BPT	6.0	0.65(11)
Clover [7]	Non-pert.	6.0	0.66(11)
TI Clover [8]	1-loop TI	$6.0, 6.2$	0.72(8)

Table 1: Lattice estimates of $B_K(NDR, \mu = 2\ \mathrm{GeV})$ for different lattice actions. An asterisk imples that the data were extrapolated to $a = 0$.

better is a matter of numerical detail, and since at this point I do not have data to make comparative statements, I direct the reader to Martinelli's writeup.

2 B_K

Even though most calculations of B_K have been done in quenched QCD, there are good reasons to believe, as discussed below, that we have a reasonable estimate for the full theory. A summary of results is presented in Table 1. The most precise calculation in terms of both statistical and systematic errors, is by the JLQCD collaboration. Their result $B_K = 0.628(42)$ when converted to the renormalization group invariant $\hat{B}_K$ is [13]

$$\hat{B}_K = 0.86(6)\,. \tag{5}$$

Since this result is for the quenched theory we have to address two associated issues. (i) Quenched chiral logs (QCL), and (ii) other effects of quenching. The other remaining systematic error is the use of degenerate quarks for the kaon. The quark mass is typically varied in the range $3m_s - m_s/3$, and the physical kaon is defined in the following two ways. (a) It is assumed to be made up of two degenerate quarks of mass $m_s/2$, or (b) the "light" quark is extrapolated to m_d and the other interpolated to m_s. The issue of quenched chiral logs is therefore relevant to (b). (The reason is that for degenerate quarks, quenched QCD and QCD have the same chiral expansion and enhanced logs due to η', an additional Goldstone boson in the quenched theory, vanish [10].) The status on each of these issues and some clarifications on the different published numbers is as follows.

- The parity even part of the 4-fermion operator has two terms, VV and AA. Sharpe [9, 10] has calculated the QCL in these and their

lattice volume dependence. The lattice data show the expected behavior, providing a basis for confidence in the CPT analyses. The leading chiral log cancels in the sum, $VV + AA$, thus alleviating the uncertainties associated with chiral extrapolations in the quenched theory.

- Estimates of quenching uncertainties provided by CPT are strengthened by the preliminary unquenched calculations [11], and suggest that $B_K(\text{full QCD}) \approx 1.05 B_K(\text{quenched})$.

- CPT has also been used by Sharpe [10] to estimate the uncertainty associated with using ($m_d \approx m_s$) rather than the physical ratio ($m_d \approx 0.055 m_s$). He estimates $B_K(\text{QCD}) \approx 1.05 \pm 0.05 B_K(\text{degenerate})$.

- Lastly, it is a fortunate coincidence that the conversion of quenched $B_K(\overline{MS}, \mu = 2\ \text{GeV})$ to $\hat{B}_K$ is very insensitive to whether one uses quenched α_s and anomalous dimensions or those for the full theory.

The success of CPT in estimating errors raises the question whether the systematic shifts due to quenching and degenerate versus physical mass quarks discussed above should be incorporated in the final value of $\hat{B}_K$ or stated as a separate error? Sharpe in [12] includes them and quotes $\hat{B}_K = 0.94 \pm 0.06 \pm 0.14$, where the second error is a very conservative estimate of the systematic uncertainties. In [13], I chose not to include them in the central value and had used a more aggressive estimate of systematic errors in quoting $\hat{B}_K = 0.86 \pm 0.06 \pm 0.06$. Both estimates are based on exactly the same data (JLQCD), and until unquenched data of comparable quality becomes available the choice between them reflects one's taste in the handling of systematic errors.

Finally, in my view, one should not average the numbers given in Table 1 to get a "best" lattice result. At present, JLQCD's is, by far, the best calculation with respect to lattice size, statistics, and systematics. (The quoted errors in Table 1 do not always include/address all the systematics uncertainties equally well). What the table does highlight is that all the results agree: a confirmation of the stability of lattice calculations of B_K.

3 Light quark masses

Since mid-1996 there has been a spurt of activity in the calculation of light quark masses from both lattice QCD and QCD sum-rules. The intriguing possibility first suggested by lattice calculations is that m_u, m_d, and m_s are much lighter than previous estimates based on QCD sum-rules. For a summary of the lattice methodology and results until Oct. 1997 see [14] and also the talk by S. Ryan [15].

	Action	$\bar{m}$	$m_s(M_K)$	$m_s(M_\phi)$	scale $1/a$
Summary 1997 [14]		3.8(1)(3)	99(3)(8)	111(7)(20)	M_ρ
APE 1998 [17]	O(a) SW	4.5(4)	111(12)		M_{K^*}
APE 1999 [18]	O(a) SW	4.1(6)	98(12)		M_{K^*}
CPPACS 1999 [19]	Wilson	4.55(18)	115(2)	143(6)	M_ρ
JLQCD 1999 [20]	Staggered	4.23(29)	106(7)	129(12)	M_ρ
ALPHA-UKQCD 1999 [21]	O(a) SW		97(4)		f_K
RIKEN-BNL 1999 [22]	DWF		95(26)		f_π
QCDSF 1999 [23]	O(a) SW	4.4(2)	105(4)		R_0
QCDSF 1999 [23]	Wilson	3.8(6)	87(15)		R_0

Table 2: Recent lattice estimates in MeV of $\bar{m}$ and m_s, in $\overline{MS}$ scheme at 2 GeV. SW stands for the Sheikhholeslami-Wohlert action.

Recent quenched lattice results are summarized in Table 2 and unlike B_K there is no single calculation that is "superior" to the rest. (Unfortunately, once again this is not obvious from quoted errors.) At first glance one sees a significant spread. Focusing attention on m_s extracted by fixing M_K to its physical value, $m_s(M_K)$, the estimates lie between $90-115$ MeV. A large part of this variation is due to the quantity used to set the lattice scale $1/a$. There is also a large difference between $m_s(M_K)$ and $m_s(M_\phi)$, *i.e.* different strange mesons give different estimates; and even though neither one reproduces the octet and decuplet baryon mass splittings, $m_s(M_\phi)$, comes much closer [16, 19]. A short explanation of the results is in order.

First, the difference between $m_s(M_K)$ and $m_s(M_\phi)$, and the variation with the observable used to fix $1/a$ are both symptoms of the quenched approximation. The data suggests that it is a $\sim 10\%$ effect, and at present constitutes the biggest uncertainty. Second, the consistency of the results using different actions (from Wilson to domain wall fermions), analyzed using the same states and after an $a = 0$ extrapolation to remove discretization errors, shows that the lattice technology is robust and that we have control over discretization errors. Third, there has been much debate over which renormalization constants (tadpole improved perturbative or from the various non-perturbation methods) to use. The data show that after extrapolation to $a = 0$, the difference is at most a few percent. So the bottom line is that we now have many different methods and consistency checks within the lattice approach for calculating light quark masses and just need the computer power to shed the last approximation – the quenched approximation – to get reliable estimates.

The only unquenched data (albeit for 2 degenerate dynamical flavors) of the modern era (using improved action, nonperturbative renormalization

constants, and extrapolation to $a = 0$) are the preliminary results by the CPPACS collaboration. T. Kaneko at LATTICE 99 reported

$$\begin{aligned} (m_u + m_d)/2 &= 3.3(4)\text{MeV} \\ m_s(M_K) &= 84(7)\text{MeV} \\ m_s(M_\phi) &= 87(11)\text{MeV} \end{aligned} \tag{6}$$

It is quite remarkable that $m_s(M_K)$ and $m_s(M_\phi)$ already show consistency. Also, the associated mass splittings in the baryon octet and decuplet are much improved. On the strength of these consistency checks I propose using

$$m_s(\overline{MS}, \mu = 2GeV) = 85(10)\text{MeV}\,. \tag{7}$$

4 $\Delta I = 3/2$ Electroweak Penguins: B_7 and B_8

Current lattice calculations of the $\Delta I = 3/2$ amplitude rely on CPT to relate $K \to \pi\pi$ to $K \to \pi$ with degenerate K and π. Under these approximations, the calculation of B_7 and B_8 is equivalent to that of B_K in complexity. Initially, there was a problem of much larger 1-loop renormalization constants. This is now under much better control through the development of better operators and non-perturbative methods. A summary of results is given in Table 3. All results using perturbative Z's are consistent, confirming that the calculation of the raw correlation functions is under control. The APE calculation [7] using non-pertubative Z's gives a value higher by $\sim 20\%$. However, since almost all the calculations have been done at only one coupling, and anticipating that the extrapolation to the $a = 0$ limit will also be different for the two ways of estimating the Z's, it is too early to choose between the two values. Calculations at other values of the coupling are in progress and I anticipate we will reduce the uncertainty to $< 10\%$ within the year.

The more important issue is whether tree-level CPT is sufficient to relate $K \to \pi\pi$ to $K \to \pi$ with $M_K = M_\pi$. Since a similar situation in B_4 suggests not [24], Golterman and Pallante are doing the needed 1-loop calculations. Thereafter, one has to confront issues of removing the quenched approximation and developing the technology for dealing with the physical case of $K \to \pi\pi$ away from threshhold in case 1-loop CPT corrections are very large. Sheer optimism propels me to believe that we will see progress towards realistic estimates of these parameters in the next couple of years.

Fermion Action	Z	β	$B_7^{3/2}$	$B_8^{3/2}$
Staggered [2]	1-loop TI	6.0, 6.2*	0.62(3)(6)	0.77(4)(4)
Wilson [5]	1-loop TI	6.0	0.58(2)(7)	0.81(3)(3)
SW [7]	1-loop BPT	6.0	0.58(2)	0.83(2)
SW [7]	Non-pert.	6.0	0.72(5)	1.03(3) .
TI SW [8]	1-loop TI	$6.0, 6.2$	0.72^{+5+2}_{-4-8}	0.80^{+8+1}_{-8-4}

Table 3: Lattice estimates of B_7, and B_8 ($NDR, \mu = 2$ GeV) for the amplitude $K \to \pi$. An asterisk implies extrapolation to $a = 0$.

5 Strong Penguin: B_6

Lattice QCD does not yet have an estimate for B_6. In addition to the issue of using CPT to relate $K \to \pi\pi$ to $K \to \pi$, there is also the problem of mixing with lower dimension operators and large renormalization factors. There are two calculations underway. One using domain wall fermions as already discussed by Blum; and the second by Kilcup and Pekurovsky using staggered fermions [25]. Kilcup *et al.* show that all the needed correlation functions can be calculated with small statistical errors, however, since the 1-loop $Z's$ for staggered fermions are very large ($\sim 100\%$) there are no reliable predictions. One definitely needs non-perturbative calculation of these. It is too early to tell if domain wall fermions will prove to be the method of choice. In short, B_6, which is crucial to understanding both the $\Delta I = 1/2$ rule and ϵ', is still an open problem in lattice QCD.

Acknowledgements

It is a pleasure to thank Bruce and Jon and for organizing such a wonderful conference, and T. Bhattacharya, C. Bernard, and S. Sharpe for their comments.

References

[1] G. Buchalla, A. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996). A. Buras, Les Houches 97, Elsevier, 1999.

[2] G. Kilcup, R. Gupta, and S. Sharpe, Phys. Rev. **D57**, 1654 (1998).

[3] S. Aoki, *et al.*, JLQCD collaboration, Phys. Rev. Lett. **80**, 5271 (1998).

[4] G. Kilcup, *et al.*, Nucl. Phys. (Proc. Suppl.) **B53**, 345 (1997).

[5] R. Gupta, T. Bhattacharya, and S. Sharpe, Phys. Rev. **D55**, 4036 (1997).

[6] S. Aoki, *et al.*, JLQCD collaboration, hep-lat/9901018.

[7] L. Conti, Phys. Lett. **421B**, 273 (1998).

[8] L. Lellouch and C.D. Lin, Nucl. Phys. (Proc. Suppl.) **B73**, 312 (1999).

[9] S. Sharpe, Phys. Rev. **D46**, 3146 (1992).

[10] S. Sharpe, TASI 94, Ed. J. Donoghue, World Scientific, 1995.

[11] G. Kilcup, *et al.*, Nucl. Phys. (Proc. Suppl.) **B53**, 345 (1997); N. Ishizuka, *et al.*, Phys. Rev. Lett. **71**, 24 (1993).

[12] S. Sharpe, Plenary talk at ICHEP98, hep-lat/9811006.

[13] R. Gupta, in "Problem of Mass" Orbis Scientiae 1997, Ed. B. Kursunoglu *et al.*, Plenum Press, 1998

[14] T. Bhattacharya, R. Gupta, Nucl. Phys. (Proc. Suppl.) **B63**, 95 (1998).

[15] S. Ryan, *these proceedings*.

[16] T. Bhattacharya, *et al.*, Phys. Rev. **D53**, 6486 (1996).

[17] D. Becirevic, *et al.*, APE Collaboration, Nucl. Phys. (Proc. Suppl.) **B73**, 222 (1999).

[18] V. Lubicz, APE Collaboration. Estimates presented at LATTICE 99 based on the method of the renormalized quark propagator.

[19] S. Aoki, *et al.*, CPPACS Collaboration, hep-lat/9904012.

[20] S. Aoki, *et al.*, JLQCD Collaboration, Phys. Rev. Lett. **82**, 4392 (1999).

[21] J. Garden, *et al.*, ALPHA-UKQCD collaboration, hep-lat/9906013.

[22] T. Blum, *et al.*, RIKEN-BNL Collaboration, hep-lat/9902016, *ibid.*

[23] M. Göckeler, *et al.*, QCDSF Collaboration, hep-lat/9908005.

[24] M. Golterman and K. Leung, Phys. Rev. **D56**, 2950 (1997).

[25] D. Pekurovsky and G. Kilcup, hep-lat/9812019